NONREFLECTING BOUNDARY CONDITIONS:
THE CONCEPT AND IMPLEMENTATION TO
NONLINEAR APPLIED PROBLEMS

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ABSTRACT

About a half of applied problems in mathematical physics are formulated for infinite spatial domains. Numerical simulation of such problems usually requires the transition to a bounded region and setting of conditions at artificial boundaries. When artificial boundary conditions yield the exact solution of the origin problem within the reduced domain specified, they are referred to as nonreflecting boundary conditions (NRBC).

For any mathematical model, we must distinguish the existence of an exact boundary condition and the ability to its construction. In this way, we clarify the notion of NRBC as a condition being exact for a family of problems described by the system of governing equations specified.

A variety of models of different type and complexity is observed from the point of view of ability to their analysis and construction of nonreflecting boundary conditions. We will concentrate at nonlinear problems. For this reason, nonlocal boundary conditions are beyond our survey, and approximate NRBC’s are considered instead. The general set of problems is illustrated with examples from gas dynamics.

For linear systems of equations, the main tool of analysis is an expansion of solutions with respect to Fourier modes or other base functions. In some cases, exact NRBC are achievable. More generally, the properties of an artificial boundary condition can be estimated by reflection coefficients.

For the 1D linearized Euler equations (more generally—nondispersive hyperbolic systems), available are exact modes and exact sets of NRBC. For the same in the 2D case, the modes are exact but there exist only approximate local NRBC’s (in the case of the Euler equations—exact convective NRBC’s). The Navier–Stokes equations (even 1D) admit only approximate modes (regular modes, i.e., corrections to the Euler modes) and complex expressions for singular modes (additional with respect to the Euler case).

Discrete (finite-difference and finite-element) models of continuous media possess dispersion error and, in most cases, parasite modes (an analogue to singular modes). Various situations are considered by several typical examples and are classified as above. NRBC are approximate with rare exceptions.

Nonlinear systems of equations admit no decomposition with respect to modes. Estimates to boundary condition errors are generally unavailable. Artificial boundary conditions are borrowed from NRBC for linearized models and implemented by special tools. High-accuracy approximate NRBC’s are of little use whereas Dirichlet boundary equations are preferable.

For the 1D nonlinear Euler equations (quasilinear hyperbolic systems nondispersive in the linearized form), exact NRBC sets exist. For the 2D nonlinear Euler equations, exact boundary conditions are not deducible. An interesting example is the 1D central-difference scheme for the 1D Euler equations where one is able to examine approximate sawtooth modes and construct approximate NRBC’s suppressing these modes. An extension to more general cases is also possible.