DETERMINATION OF THE HEAT TRANSFER COEFFICIENT DISTRIBUTION ON THE LONGITUDINAL FINNED TUBES IN STAGGERED ARRANGEMENT USING INVERSE AND CFD METHOD

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Abstract. This paper presents results of an experimental study on heat exchanger consisting of finned tube. The presented method for determining the local heat transfer coefficient on external tube surfaces is characterized by very high accuracy and can be applied to determine the spatial heat transfer distribution on objects with a complex shape. As a results of experimental and numerical investigations were obtained distribution of the local heat transfer coefficient on the surface of the finned tube for various Reynolds numbers and the temperature distribution on their surfaces too.

1 INTRODUCTION

Local heat transfer from a cylinder in tube bank has been extensively studied. In many applications such as design of heat exchangers, detailed information regarding the circumferential and longitudinal variation of heat transfer to a cylinder is required. There are many different methods for measuring local heat transfer like techniques using liquid crystals, thermal paints, heater foils, naphthalene heat-mass transfer analogy [1-5]. All techniques have been widely used with considerable success but they have certain difficulties.

An alternative method to obtain the local convective heat transfer coefficient is the inverse procedure. This paper presents results of an experimental study on heat exchanger consisting of finned tube. Tubes have two longitudinal fins which join neighboring tubes, creating membrane panels. The local and mean heat transfer coefficients on the tube circumference were determined using the inverse heat transfer method. Using the inverse methods [6,7], the convective heat-transfer coefficient was determined very exactly, reproducing the real two-dimensional temperature distribution in the cross-section of the tube. In this way, the circumferential heat flow in the tube caused by the non-uniform heat-transfer coefficient was taken into consideration. In many past examinations that phenomenon was disregarded, assuming heat flow in the tube in radial direction only. Such an incorrect assumption can lead to incorrectly determining heat transfer coefficients, which can differ from real values by even
Determination of the space-variable heat transfer coefficient on a complex shape surface requires the solution of the nonlinear inverse heat conduction problem \([8-10]\). The unknown parameters associated with the solution are selected to achieve the closest agreement in a least squares sense between the computed and measured temperatures using the Gauss-Newton method in conjunction with the singular value decomposition or modified Gram-Schmidt methods. Hensel and Hills \([6]\) approached the two-dimensional steady-state inverse heat conduction problem using the linear least-squares method. Linearization of the least squares problem is accomplished by assuming unknown temperatures \([6]\) or temperatures and heat fluxes \([7, 8]\) on the boundary.

The boundary is divided into a large number of elements, and temperatures or heat fluxes are assumed to be constant over each element. Having determined the boundary values of temperature and heat flux from the solution of the IHCP, the convective heat transfer coefficients are determined from Newton’s Law of Cooling. Numerical \([6,7]\) and experimental tests demonstrated that spatial distribution of the heat transfer coefficient can be estimated with satisfactory accuracy if the division of the boundary into elements is very fine. If the number of segments on the boundary is too small, then the constant value of temperature or heat flux over an element cannot be assumed. In order to solve over-determined IHCP the number of interior temperature measurement points should be greater than the number of components of boundary heat transfer coefficient or than the number of unknown parameters. In this paper techniques are considered in which the distribution of heat transfer coefficient is deduced from internal temperature measurements. The parameters in the function describing the spatial variation of the heat transfer are investigated. The thermal conductivity of the solid \(k(T)\) may be temperature-dependent.

The experimental results reported herein are among the first that show the variation of the local heat transfer coefficients over the circumference of the finned tube. Most data reported previously were acquired for smooth tubes at low temperatures. The main advantage of the method is that it does not require any knowledge of, or solution to, the complex fluid flow field. It should be noted that determining unknown steady distribution of heat transfer coefficients by using the developed method is inexpensive, since it requires only one fluid temperature probe and a few thermocouples for temperature measurements inside the solid.

2 MATHEMATICAL FORMULATION OF THE INVERSE HEAT CONDUCTION

The temperature distribution in the body is governed by nonlinear partial differential equation

\[
\nabla \cdot [k(T) \nabla T] = 0. 
\]

The unknown boundary condition of the third kind is prescribed on the outer surface of the body (Fig. 1)

\[
k(T) \frac{\partial T}{\partial n} = h(r_s)(T_e - T)
\]

where \(r_s\) represents points on the boundary, \(s\).
In addition to the unknown boundary conditions the internal temperature $f_i$ measurements are included in the analysis:

\[ T(r_i) = f_i, \quad i = 1, \ldots, m, \quad m \geq n \]  

The objective of the present approach is to determine the spatial distribution of the heat transfer coefficient $h(s)$ based on measured temperatures at $m$ interior locations. The way of solving this problem is presented in the paper. In the approach, the problem of determining space-variable heat transfer coefficient is formulated as a parameter estimation problem by selecting the functional form $h = h(s, x_1, \ldots, x_n)$ for the heat transfer coefficient. In this case parameters $x_1, \ldots, x_n$ have no physical meaning. There are $n$ parameters in $x = (x_1, \ldots, x_n)^T$ to be determined such that the computed temperatures $T_i$ agree with the experimentally acquired temperatures $f_i, \ i = 1, \ldots, m$ in the least-squares sense. A standard procedure is to take more temperature measurements than the number of unknown parameters $x_i$. The least-squares method is used to determine $x_1, \ldots, x_n$ when $m > n$. To measure how well the calculated temperature agree with data, the sum of squares is used

\[ S = (f - T_n)^T (f - T_n). \]  

The Levenberg-Marquardt method [11] is used to determine $x^*$ for which the sum $S$ becomes minimum.

The method performs the $k$-th iteration as

\[ x^{(k+1)} = x^{(k)} + \delta^{(k)}, \]  

where
The value of the parameter $\mu^{(k)} \to 0$ when $x^{(k)} \to x^*$. In the proximity of minimum $x^*$ the iteration step in the Levenberg-Marquardt method is almost the same as in the Gauss-Newton method. The computation programs for solving the non-linear least squares problem by the Levenberg-Marquardt method are described in [13] and in the IMSL Library [12].

3 EXPERIMENTAL APPARATUS AND TECHNIQUE

In the present experiment a wind tunnel of 270 x 300 mm cross-section and a radial fan is used. A bank of finned tubes (called membrane panels) in staggered arrangement is schematically shown in Fig. 2. The fan output is controlled by an inverter. In the first part of the aerodynamic channel, there is a chamber in which the heating banks are placed. Each bank consists of 36 tubes made of K18 boiler steel, having a length of 300 mm and a 24.70 mm outer diameter. The tubes are arranged in staggered order, in rows of five tubes each, with a transverse pitch $S_1 = 53.25$ mm and longitudinal pitch $S_2 = 31.00$ mm. Each tube was fitted with a heating element supplied from 380 V AC mains. The stand was arranged with measuring instruments for air temperature, static and dynamic pressure, tube wall temperature, and power consumed by the heating elements. The temperature at the inlet $T_{in}$ and outlet $T_{out}$ of the tube bank was measured by NiCr-Ni thermocouples. The central tube was fitted at the middle of its height with 9 thermocouples (Fig. 2b). They supplied data for recording the temperature of the tube surface. The UPN-100 data-logger is provided with software for data acquisition by an PC computer. Heat loses from the duct and heated tube bundle were minimised by insulating the outside surface of the duct with fiberglass mats. The heat transfer coefficient on the outer surface of the finned tube was determined by measuring the electric power $\dot{Q}_e$ supplied to the heater placed inside the tube of the inner diameter $d_{in}$ and the height $h_t$. The heat transfer rate generated by the electric heater at the interior of the investigated tube is transferred to the air through its the inner and outer surfaces. The heat flux at the tube inner surface is not uniform due to circumferential heat conduction at the tube wall.
The same heat flow rate is generated at each tube row. Hence, the air temperature changes along its flow path can be assumed as linear. The air temperature flowing around the investigated tube (Fig. 2b) was computed using the formula

$$T_a = \frac{(L_b - L_t)}{L_b} T_{in} + \frac{L_t}{L_b} T_{out} = \frac{(215 - 93)}{215} T_{in} + \frac{93}{215} T_{out} = 0.5674 T_{in} + 0.4326 T_{out},$$

where

$L_b$ – bundle length, m,
$L_t$ – distance of the investigated tube from the bundle inlet, m.

The local heat transfer coefficients were determined on the basis of the balance of heat energy supplied by the heating elements placed in the tubes and the power absorbed by air flowing around the exchanger tubes surface. The heat flux on the internal tube surface is calculated from the equation

$$\dot{q} = \frac{\dot{Q}}{n d_m h_i}.$$  

The temperature distribution in the tube cross section was calculated using the control volume method. Owing to the symmetry of the system, the calculus was restricted to one half of the tube cross section. That half of the cross section was divided into control volumes, for which 75 heat balance equations were obtained. The distribution of the convective heat transfer coefficient $h$ was determined by the Levenberg-Marquardt method, using nine temperature values measured on the finned tube. The thermocouples were located 0.5 mm under the tube surface, at the nodes no. 1, 4, 7, 9, 11, 13, 15, 18, 21 (Fig. 3). The distribution of the local convective heat transfer coefficient on the surface of the finned tube was
approximated by the function:

\[ h(H) = x_0 + \sum_{i=1}^{4} x_i \cdot \cos \left( i \cdot \frac{H}{H_c} \pi \right) \]  \hspace{1cm} (10)

\[ H \] – coordinate, measured from point 1, m

\[ H_c \] – extended length of tube semi-circumference between points 1 and 21 (Fig. 3), m

At each iteration step the set of balance equations system of control volumes was solved using the Gauss elimination method.

3 EXPERIMENTAL RESULTS

The measured and calculated temperature distribution on the fined tube semicircumference for the two values of the Reynolds number is shown in Fig. 4. The axis of abscissae represent half length of developed circumference measured from point 1 to 21 (Fig. 3) in a direction of flow. The length of developed semicircumference is \( H_c = 73.29 \) mm. The changes of the heat transfer coefficient for various Reynolds numbers are depicted in Fig. 5. It follows from Fig. 5 that high values of \( h \) heat transfer coefficient occur at the front of the fin. It decreases gradually since it reaches first minimum at the base of fin. Heat transfer results show a peak on cylindrical part at point \( \phi = 90^\circ \). Second minimum occur at the base of rear fin. At these locations a large reduction in \( h \) is due the wakes. Flow in the wake region is characterised by low air velocities what corresponds to the low value of heat transfer coefficient.
4 NUMERICAL RESULTS

In order to validate the inverse method for obtaining local heat transfer coefficient, numerical simulations of the flow in the heat exchanger which consists of finned tube were carried out. Numerical simulations were conducted using commercial software Fluent. Because of the symmetry of the tube bank geometry, only a portion of the domain needs to be modeled. The computational domain is shown in Figure 6. The temperature and velocity are obtained Fig.7. The comparison of the distribution of the local heat transfer coefficient
obtained from the numerical simulation using the program Fluent [14] and from the inverse method is shown in Fig. 8.

From an inspection of the results shown in Fig. 8 it follows that both approaches give similar results. The distributions of the heat transfer coefficient depicted in Fig. 8a and 8b show similar behaviour, although noticeable differences are observed in the values of $h(\phi)$. The choice of a turbulence model has remarkable influence on the obtained distribution $h(\phi)$ and the mean value $\overline{h}$ of the heat transfer coefficient (Table 1). The value of $\overline{h}_F = 57.7$ W/(m$^2$K) obtained from numerical calculation using k-\(\varepsilon\) turbulence model is close to the value $\overline{h} = 54.8$ W/(m$^2$K) obtained from the inverse method.
Figure 8: Distribution of heat transfer coefficient on membrane panel surface for Re = 33000
a) numerical simulation, k-ε turbulence model, $h_F = 139.9 \text{ W/(m}^2\text{K)}$, b) inverse method $h = 125.8 \text{ W/(m}^2\text{K)}$

<table>
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<th>Mesh</th>
<th>Turbulence model</th>
<th>Heat transfer coefficient $h_F$, W/(m$^2$K) (Fluent)</th>
<th>Heat transfer coefficient $\bar{h}$, W/(m$^2$K) (Measurement)</th>
<th>$\frac{h_F - \bar{h}}{\bar{h}}$</th>
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<td></td>
<td>k-ε</td>
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<td>-10.0</td>
<td>18.2</td>
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Table 1: Computation results for the membrane surface heat exchanger; $w = 3.6 \text{ m/s, } T_{in} = 22.4 \degree \text{C, } q = 5080 \text{ W/m}^2, \text{Tu}=3\%$

S-A – Spalart-Allmaras turbulence model,
k-ε – k-ε turbulence model,
RSM – Reynolds stress turbulence model

5 CONCLUSION

The results of an experimental studies and numerical calculations of the finned tube heat exchanger were presented in the paper. The experimental data were obtained from the measurements of the finned tube temperature on the outer surface, air velocity and heat on the inner surface of the tube. The distribution of local heat transfer coefficient on the outer surface of the finned tube was approximated by the trigonometric polynomial of the fourth
kind. And the local heat transfer coefficient was calculated using least squares method with Levenberg-Marquardt algorithm. The average heat transfer coefficient was calculated using CFD software – FLUENT. The comparison of the local and average heat transfer coefficient was shown.

REFERENCES


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